

A LITTLE BIT OF HISTORY

The Babylonians (about 400 BC) appear to be the first to solve quadratic equations.

The Hindu mathematician Brahmagupta (598-665 AD) essentially used the quadratic formula to solve quadratics. Brahmagupta used letters for variables and considered negative numbers.

al-Khwarizmi (c 820 AD) also essentially used the quadratic formula to solve quadratics but didn't consider negative solutions.

Abraham bar Hiyya Ha-Nasi was the first to publish a book in 1145 which used the complete quadratic formula.

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Number and Algebra ➔ Linear and non-linear relationships

★ Solve linear equations involving simple algebraic fractions.

- solve a wide range of linear equations, including those involving one or two simple algebraic fractions, and check solutions by substitution.
- represent word problems, including those involving fractions, as equations and solve them to answer the question.

★ Solve simple quadratic equations using a range of strategies.

- use a variety of techniques to solve quadratic equations, including grouping, completing the square, the quadratic formula and choosing two integers with the required product and sum.

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A TASK

A graphics calculator can be used to solve quadratic equations.

Learn how to use a graphics calculator to solve quadratic equations such as:

\[6x^2 + 7x - 5 = 0\]

The solutions are \(x=0.5\), and \(x = -1.67\)

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Quadratic - the highest power of the variable is two.

\[u^2 + 2P = 2h\]

Air pressure and the Bernoulli effect.

\[s = ut + \frac{1}{2}at^2\]

Distance travelled by accelerating object.

\[\frac{\partial u}{\partial t} + \nabla^2 u + v(x)u = 0\]

Design of integrated circuits in mobile phones and computers.
Solving Linear Equations

To solve an equation is to find the value of the unknown number (the variable) in the equation.

Exercise 7.1

Solve each of the following equations:

\[
\begin{align*}
\text{1} & \quad x + 8 = 3 \\
\text{2} & \quad b - 7 = -11 \\
\text{3} & \quad 5y = 25 \\
\text{4} & \quad t \div 4 = 3 \\
\text{5} & \quad a + 11 = 43 \\
\text{6} & \quad 21m = 105 \\
\text{7} & \quad z \div 7 = 11 \\
\text{8} & \quad -2w = 48 \\
\text{9} & \quad x - 29 = 41 \\
\text{10} & \quad 3(x + 4) = 21 \\
\text{11} & \quad -6(m - 2) = 18 \\
\text{12} & \quad 2(a - 3) = 4 \\
\text{13} & \quad 4(x + 2) = 16 \\
\text{14} & \quad -2(f + 5) = 10 \\
\text{15} & \quad 5(a + 6) = 25 \\
\text{16} & \quad 7(x - 3) - 12 = 28 \\
\text{17} & \quad 3(-3x + 2) = 35 \\
\text{18} & \quad 3(5x - 4) = -15 \\
\text{19} & \quad 5(\text{a} + 6) = 25 \\
\text{20} & \quad 5(\text{a} + 6) = 25 \\
\text{21} & \quad 2(2x - 2) = 14 \\
\text{22} & \quad 3(x + 2) + 2x = 11 \\
\text{23} & \quad \text{If the unknown occurs more than once, put the unknowns together (5x-3x=2x).}
\end{align*}
\]

The inverse of  + is  −
The inverse of  − is  +
The inverse of  × is  ÷
The inverse of  ÷ is  ×
### Solving Linear Equations

#### Chapter 7 Solving Equations

#### Solving Linear Equations

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<td><strong>Check:</strong> $(0.36)^2 + 8 \times 0.36 - 3 = 0$</td>
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#### If the unknown is on both sides of the $=$, put the unknowns together.

#### Multiply by the denominator of the fraction.

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<td>$a/5 - 2 = 4$</td>
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<td>$y/3 - 3 = -1$</td>
<td>$y = -6$</td>
</tr>
<tr>
<td>$x/5 - 15 = -3$</td>
<td>$x = -30$</td>
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<td>$2(2x+1) = 3x - 24$</td>
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<td>$4x + 2 = 3x - 24$</td>
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<td>$x = 26$</td>
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<td><strong>Check:</strong> $(2 \times 26 + 1)/3 = -26/2 - 4$</td>
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Each day, in Australia, millions of real-world problems are solved by the use of linear equations.

The basic idea is to:
1 Write the formula.
2 Substitute.
3 Solve for the unknown.

Exercise 7.2

The circumference, C, of a circle is given by the formula: \( C = 2\pi r \), where \( r \) is the radius of the circle. A circular horse yard is to have a circumference of 56 m. What should be the radius of the horse yard?

\[
C = 2\pi r \quad \{\text{write the formula}\}
\]

\[
56 = 2\pi r \quad \{\text{substitute } C = 56\}
\]

\[
\frac{56}{(2\pi)} = r \quad \{\text{inverse of } \times \text{ is } \div\}
\]

\[
8.91 = r
\]

Radius of horse yard = 8.91 m.

Check: \(2\pi \times 8.91 = 55.98 \text{ m}\)

1 The circumference, \( C \), of a circle is given by the formula: \( C = 2\pi \), where \( r \) is the radius of the circle. A circular horse yard is to have a circumference of 75 m. What should be the radius of the horse yard?

2 The perimeter, \( P \), of a rectangle is given by \( P = 2(l + b) \), where \( l \) is the length and \( b \) is the breadth. If the length of a house block is 43 m and the perimeter is 138 m, what is the breadth of the house block?

3 Speed, \( v \), is given by the formula: \( v = \frac{s}{t} \) \((v = \frac{s}{t})\), where \( s \) is the distance and \( t \) is the time. If thunder is heard eight seconds after the lightning is seen, how far away was the lightning (Assume sound travels at 330 m/s)?

4 The sum of the interior angles of a polygon is given by the formula: \( S = 90(2n - 4) \), where \( n \) is the number of sides on the polygon. How many sides in a polygon with an interior angle sum of 540°?

5 The volume, \( V \), of a square based prism is given by the formula: \( V = w^2h \), where \( w \) is the width of the base and \( h \) is the height of the prism. If the width of the prism is 4 cm and the volume is 96 cm\(^3\), what is the height of the prism?

6 The volume of a cylinder, \( V \), is given by the formula: \( V = \pi r^2h \), where \( r \) is the radius of the base of the cylinder and \( h \) is the height of the cylinder. If a cylinder with a base radius of 6.5 cm has a volume of 1487 cm\(^3\), what is the height of the cylinder?

7 The volume of a cone, a circular based pyramid, is given by the formula: \( V = \frac{1}{3}\pi r^2h \) where \( r \) is the radius of the base of the cone and \( h \) is the height of the cone. If a cone has a radius of 1.1 m and a volume of 6.8 m\(^3\), what is the height of the cone?
Exercise 7.3

Five times a number increased by four is divided by three to obtain twenty-two. What is the number?

Let the number be \( x \),

\[
\frac{5x + 4}{3} = 22
\]

\[
5x + 4 = 22 \times 3
\]

\[
5x + 4 = 66
\]

\[
x = 66 - 4
\]

\[
x = 62
\]

\[
x = 62 \div 5
\]

\[
x = 12.4
\]

Check: \((5 \times 12.4 + 4)/3 = 66/3 = 22\)

A rectangle has a length which is 5 m less than twice its width. What is the width of the rectangle, if the perimeter is 80 m?

Let the width be \( w \),

\[
2w + 2(w - 5) = 80
\]

\[
2w + 2w - 10 = 80
\]

\[
4w - 10 = 80
\]

\[
4w = 80 + 10
\]

\[
4w = 90
\]

\[
w = 90 \div 4
\]

\[
w = 22.5 \text{ m (the width)}
\]

Check: \(2 \times 22.5 + 2(22.5 - 5) = 80\)

The crocodile was described as being one-third tail, one-quarter head and with a 300 cm body. What was the length of the crocodile?

Let the length of the crocodile be \( x \),

\[
\frac{1}{3}x + \frac{1}{4}x + 300 = x
\]

\[
4x + 3x + 3600 = 12x \quad \{\times 12\}
\]

\[
7x + 3600 = 12x
\]

\[
3600 = 12x - 7x
\]

\[
3600 = 5x
\]

\[
3600 \div 5 = x
\]

\[
x = 720 \text{ (crocodile was 7.2 m)}
\]

Check: \(720/3 + 720/4 + 300 = 720\)

1. Three times a number increased by five is divided by two to obtain thirteen. What is the number?
2. Four times a number decreased by six is divided by seven to obtain twenty-two. What is the number?
3. Three consecutive numbers have a sum of forty-five. What are the numbers (4, 5, and 6 are consecutive numbers)?
4. Find three consecutive numbers whose sum is six times the first number.
5. A rectangle has a length which is 3 m less than twice its width. What is the width of the rectangle, if the perimeter is 42 m?
6. A rectangle has a length twice the size of the width. What is the width of the rectangle if the perimeter is 180 cm?
7. An isosceles triangle has two angles each four times the size of the third angle. What is the size of each angle?
8. The crocodile was described as being one-third tail, one-quarter head and with a 200 cm body. What was the length of the crocodile?
9. The first side of a triangle is 20 cm longer that the second side. The third side is half the length of the second side. What is the length of the second side if the perimeter is 300 cm?
10. The difference between two numbers is 29. One number is 5 less than three times the other. What are the two numbers?
### Exercise 7.4
Solve the following quadratics:

<table>
<thead>
<tr>
<th>Equation</th>
<th>Solution</th>
<th>Check</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^2 + 6x + 9 = 0$</td>
<td>$x = -3$</td>
<td>$(\text{-}3)^2 + 6 \times \text{-}3 + 9 = 0$ ✓</td>
</tr>
<tr>
<td>$(x + 3)^2 = 0$</td>
<td>$x = -3$</td>
<td>✓</td>
</tr>
<tr>
<td>$x + 3 = 0$</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td><strong>Check:</strong> $(\text{-}3)^2 + 6 \times \text{-}3 + 9 = 0$ ✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x^2 - 8x + 16 = 0$</td>
<td>$x = 4$</td>
<td>$(\text{-}4)^2 - 8 \times \text{-}4 + 16 = 0$ ✓</td>
</tr>
<tr>
<td>$(x - 4)^2 = 0$</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>$x - 4 = 0$</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td><strong>Check:</strong> $(\text{-}4)^2 - 8 \times \text{-}4 + 16 = 0$ ✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$4x^2 + 12x + 9 = 0$</td>
<td>$x = -\frac{3}{2}, x = -3$</td>
<td>$4 \times (-\frac{1.5}{2})^2 + 12 \times (-\frac{1.5}{2}) + 9 = 0$ ✓</td>
</tr>
<tr>
<td>$(2x)^2 + 2 \times 2x \times 3 + (3)^2 = 0$</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>$(2x + 3)^2 = 0$</td>
<td>$x = -\frac{3}{2}$</td>
<td>✓</td>
</tr>
<tr>
<td>$2x + 3 = 0$</td>
<td>$x = -\frac{3}{2}$</td>
<td>✓</td>
</tr>
<tr>
<td>$2x = -3$</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td><strong>Check:</strong> $4 \times (-1.5)^2 + 12 \times (-1.5) + 9 = 0$ ✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x^2 + 6x + 8 = 0$</td>
<td>$x = -4$</td>
<td>$x^2 + 6x + 8 = 0$ ✓</td>
</tr>
<tr>
<td>$x^2 + 4x + 2x + 8 = 0$</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>$x(x + 4) + 2(x + 4) = 0$</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>$(x + 4)(x + 2) = 0$</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td><strong>Check:</strong> $(\text{-}4)^2 + 6 \times \text{-}4 + 8 = 0$ ✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Check:</strong> $(\text{-}2)^2 + 6 \times \text{-}2 + 8 = 0$ ✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x^2 - 3x - 10 = 0$</td>
<td>$x = 5, x = -2$</td>
<td>$x^2 - 3x - 10 = 0$ ✓</td>
</tr>
<tr>
<td>$x^2 - 5x + 2x - 10 = 0$</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>$x(x - 5) + 2(x - 5) = 0$</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>$(x - 5)(x + 2) = 0$</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td><strong>Check:</strong> $(\text{-}5)^2 - 4 \times 5 - 10 = 0$ ✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Check:</strong> $(\text{-}2)^2 - 4 \times \text{-}2 - 10 = 0$ ✓</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Exercise 7.5
Solve the following quadratics by completing the square:

1. \( x^2 + 6x - 1 = 0 \)
2. \( x^2 + 2x - 3 = 0 \)
3. \( x^2 + 4x - 4 = 0 \)
4. \( x^2 + 4x - 6 = 0 \)
5. \( x^2 + 6x - 2 = 0 \)
6. \( x^2 + 10x - 5 = 0 \)
7. \( x^2 + 2x + 1 = 0 \)
8. \( x^2 + 6x + 4 = 0 \)
9. \( x^2 + 8x + 3 = 0 \)
10. \( x^2 + 12x + 2 = 0 \)

11. \( x^2 - 2x - 3 = 0 \)
12. \( x^2 - 4x - 1 = 0 \)
13. \( x^2 - 6x - 5 = 0 \)
14. \( x^2 - 8x - 7 = 0 \)
15. \( x^2 - 10x - 2 = 0 \)
16. \( x^2 - 12x - 4 = 0 \)
17. \( x^2 - 14x + 4 = 0 \)
18. \( x^2 - 6x + 2 = 0 \)
19. \( x^2 - 4x + 1 = 0 \)
20. \( x^2 - 8x + 3 = 0 \)

21. \( x^2 - 3x - 1 = 0 \)
22. \( x^2 + 3x - 3 = 0 \)
23. \( x^2 + 5x - 5 = 0 \)
24. \( x^2 - 5x - 2 = 0 \)
25. \( x^2 + 7x - 4 = 0 \)
26. \( x^2 - 7x - 3 = 0 \)
27. \( x^2 + 9x + 1 = 0 \)
28. \( x^2 - 9x + 2 = 0 \)
29. \( x^2 + 11x + 5 = 0 \)
30. \( x^2 - 11x + 2 = 0 \)
The Quadratic Formula

Thousands of problems can be expressed as quadratics. Quadratics can be solved by guess and check, sketching graphs, factorising, and using the quadratic formula.

Given the quadratic: 
\[ ax^2 + bx + c = 0 \]

The solution is:
\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

Exercise 7.6
Use the quadratic formula to solve the following quadratics:

1. \[ x^2 + x - 2 = 0 \]
   - Write the quadratic \[ x^2 + x - 2 = 0 \]
   - Write the general quadratic \[ ax^2 + bx + c = 0 \]
   - Decide values of a, b, c \[ a=1 \quad b=1 \quad c=-2 \]
   - Write the quadratic formula \[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
   - Substitute for a, b, and c \[ x = \frac{-1 \pm \sqrt{1^2 - 4 \times 1 \times (-2)}}{2 \times 1} \]
   - Calculate \[ x = \frac{-1 \pm \sqrt{1 + 8}}{2} \]
   - \[ x = \frac{-1 \pm \sqrt{9}}{2} \]
   - \[ x = \frac{-1 \pm 3}{2} \]
   - \[ x = \frac{-1 + 3}{2} \quad \text{or} \quad x = \frac{-1 - 3}{2} \]
   - Check: \( x = 1 \), \( 1^2 + 1 - 2 = 0 \) √
   - Check: \( x = -2 \), \((-2)^2 + (-2) - 2 = 0 \) \( -4/4 = 0 \) √

The numerator needs brackets if using a calculator.

1. \[ x^2 - x - 2 = 0 \]
   - which is the same as \( (x - 2)(x + 1) = 0 \)
2. \[ x^2 + x - 2 = 0 \]
   - which is the same as \( (x - 1)(x + 2) = 0 \)
3. \[ x^2 - 2x - 3 = 0 \]
   - which is the same as \( (x + 1)(x - 3) = 0 \)
4. \[ x^2 + 2x + 1 = 0 \]
   - which is the same as \( (x + 1)(x + 1) = 0 \)
5. \[ x^2 + 3x + 2 = 0 \]
   - which is the same as \( (x + 2)(x + 1) = 0 \)
6. \[ x^2 + 5x + 6 = 0 \]
   - which is the same as \( (x + 2)(x + 3) = 0 \)
7. \[ x^2 + 4x + 3 = 0 \]
   - which is the same as \( (x + 1)(x + 3) = 0 \)
8. \[ x^2 - 2x + 1 = 0 \]
   - which is the same as \( (x - 1)(x - 1) = 0 \)
9. \[ x^2 - 3x + 2 = 0 \]
   - which is the same as \( (x - 1)(x - 2) = 0 \)
10. \[ x^2 - 5x + 6 = 0 \]
    - which is the same as \( (x - 2)(x - 3) = 0 \)
**Exercise 7.7**

Use the quadratic formula to solve the following quadratics:

<table>
<thead>
<tr>
<th>Equation</th>
<th>Solution</th>
</tr>
</thead>
</table>
| \[5x^2 - 2x - 4 = 0\] | \[x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \cdot 5 \cdot (-4)}}{2 \cdot 5}\]  
  \[x = \frac{2 \pm 9.17}{10}\]  
  \[x = \frac{2 + 9.17}{10}\] or \[x = \frac{2 - 9.17}{10}\]  
  \[x = 1.12\] or \[x = -0.72\] |

**Check:**  
\[5 \times 1.12^2 - 2 \times 1.12 - 4 = 0\]  
\[5 \times (-0.72)^2 - 2 \times (-0.72) - 4 = 0\]

---

1. \[2x^2 + 3x - 2 = 0\]
2. \[x^2 + 2x - 5 = 0\]
3. \[5x^2 - 2x - 1 = 0\]
4. \[4x^2 - 3x - 3 = 0\]
5. \[2x^2 + x - 2 = 0\]
6. \[x^2 - 3x - 5 = 0\]
7. \[x^2 + 5x + 1 = 0\]
8. \[7x^2 - 2x - 5 = 0\]
9. \[4x^2 + 8x + 3 = 0\]
10. \[x^2 - 11x - 1 = 0\]
11. \[6x^2 + 8x - 4 = 0\]
12. \[4x^2 - 5x = 0\]
13. \[x^2 + 3x + 2 = 0\]
14. \[4x^2 - 8 = 0\]
15. \[8x^2 + 3x = 0\]
16. \[5x^2 - 4x = 0\]
17. \[16x^2 - 4 = 0\]
18. \[14x^2 - 5 = 0\]
19. \[x^2 + 5x - 2 = 0\]
20. \[4x^2 - 3x + 1 = 0\]
21. \[2x^2 + 3x + 5 = 0\]
22. \[3x^2 + 3x + 1 = 0\]
23. \[x^2 - 4x - 1 = 0\]
24. \[3x^2 - 6x - 2 = 0\]
25. A stone is thrown vertically with a velocity of 30 m/s.  
   The motion of the stone is described by the relationship: \[5t^2 - 30t + h = 0\], where \(h\) is the height in metres and \(t\) is the time in seconds.  
   How long will it take the stone to reach a height of 20 m?  
26. Calculate \(x\) in each of the following figures:
Mental Computation

Exercise 7.8
1 Spell Quadratic.
2 What is the quadratic formula?
3 \(3x^2 + 2x - 2 = 0\). What are the values of a, b, and c?
4 Expand \((x - 2)^2\)
5 Factorise \(x^2 + 4x + 3\)
6 What is the value of: \(\log_{10} 100\)
7 Solve: \(x + y = 10, \ xy = 21\)
8 \(2 - 5\)
9 \((x^{-3})^4\)
10 Increase $6 by 30%

Exercise 7.9
1 Spell ‘Solving linear equations’.
2 What is the quadratic formula?
3 \(5x^2 - x - 3 = 0\). What are the values of a, b, and c?
4 Expand \((x + 2)^2\)
5 Factorise \(x^2 + 7x + 10\)
6 What is the value of: \(\log_{10} 1000\)
7 Solve: \(x + y = 11, \ xy = 30\)
8 \(2 - 3\)
9 \((x^{-3})^4\)
10 Increase $7 by 30%

Exercise 7.10
1 Spell 'Completing the square'.
2 What is the quadratic formula?
3 \(x^2 - 2x + 4 = 0\). What are the values of a, b, and c?
4 Expand \((x - 3)^2\)
5 Factorise \(x^2 + 5x + 4\)
6 What is the value of: \(\log_{10} 1000\)
7 Solve: \(x + y = 6, \ xy = 8\)
8 \(7 + 7\)
9 \((x^{-3})^5\)
10 Increase $8 by 30%

Mental computation gives you practice in thinking.

\[x^2 + 4x + 3\]
\[3 + 1 = 4\]
\[3 \times 1 = 3\]
\[= (x+3)(x+1)\]

Learning without thought is labor lost; thought without learning is perilous - Confucius.

War does not determine who is right, only who is left - a paraprosdokian.

You do not need a parachute to skydive. You only need a parachute to skydive twice - another paraprosdokian.

Mechanical engineers design and supervise the construction and operation of machinery.
- Relevant school subjects are English, Mathematics, and Physics.
- Courses usually involve an engineering degree.
Exercise 7.11

If $48a = b^2$ and $a$ and $b$ are positive integers, find the smallest value of $a$.

$48a = b^2$
$16 \times 3a = b^2$
$4^2 \times 3a = b^2$  thus $a = 3$

If $a$ and $b$ are positive integers, find the smallest value of $a$:

1. $28a = b^2$
2. $24a = b^2$
3. $63a = b^2$

What is the value of $x$ when $y = -3$?

$y = \frac{3x - 5}{2}$

$-3 \times 2 = 3x - 5$
$-6 + 5 = 3x$
$-1 + 3 = x$
$x = \frac{-1}{3}$ or $0.33$

What is the value of $x$ when $y = -2$?

4. $y = 4x + 3$
5. $y = \frac{2x}{5} + \frac{1}{3}$
6. $y = \frac{4x + 3}{5}$

Solve each of the following quadratic equations:

7. $(x + 1)(x - 1) = 0$
8. $(x - 2)(2x - 1) = 0$
9. $(3x + 1)(x - 2) = 0$
10. $(2x - 2)(x + 4) = 0$

11. What is the equation of each of the following quadratics:

- a)
- b)
Exercise 7.12
1. On Monday a plant is 2 cm high. Each day the plant doubles its height from the day before. How high will the plant be on Friday?

2. Can you cut this shape into
   a) Two congruent shapes?
   b) Three congruent shapes?
   c) Four congruent shapes?
   d) Six congruent shapes?

A Game

**Calculator Hi Lo** is played on a calculator. You try to guess the secret number in as few guesses as possible.

1. Someone else gives the limits of the secret number eg., 1 to 10, 1 to 100, 1 to 1000 and enters the secret number in the calculator's memory and clears the display.

2. You enter a guess on the calculator.

3. Someone else divides your number by the number in memory.

4. If the display is:
   - greater than 1 then the guess was too high. 13.867321
   - less than 1 then the guess was too low. 0.67567567
   - equal to 1 then the guess was correct.

Try to guess the secret number in as few guesses as possible.

A Sweet Trick

Impress your audience by showing that you know some tricky patterns.

- **Think about your presentation:**
  - Think up some story about these numbers?
  - Audience has calculators, you don't have a calculator?

- **Examples:**
  - $37 \times 3 = 9^2 - 2^2$
  - $37 \times 6 = 89^2 - 12^2$
  - $37 \times 9 = 889^2 - 112^2$
  - $37 \times 12 = 8889^2 - 1112^2$
  - $37 \times 15 = 88889^2 - 11112^2$
  - $1 \times 8 + 1 = 6^2 - 5^2$
  - $12 \times 8 + 2 = 56^2 - 45^2$
  - $123 \times 8 + 3 = 556^2 - 445^2$
  - $1234 \times 8 + 4 = 5556^2 - 4445^2$
  - $12345 \times 8 + 5 = 55556^2 - 44445^2$
Investigations

Investigation 7.1 A quadratic function

The distance an object falls in a certain time, when dropped, can be modelled by the quadratic function: 
\[ s = 4.9t^2, \]
where \( s \) is the distance in metres an object falls in \( t \) seconds.

Example: A stone takes 2.75 seconds to reach the bottom of a well.

\[ s = 4.9t^2 \]
\[ \text{Depth of well} = 4.9 \times 2.75^2 \]
\[ = 37 \text{ m} \]

Investigation 7.2 A linear function?

1. Turn on a tap and let the water run at a constant rate.
2. Ready the bucket by putting a ruler inside the bucket.
3. Put the bucket under the tap and record the height of water every 10 seconds (You may wish to record every 30 s or so dependent upon how much water is flowing from the tap).
4. Draw a graph.

Two statisticians were flying from Perth to Sydney. About an hour into the flight, the pilot announced, 'Unfortunately, we have lost an engine, but don’t worry: There are three engines left. However, instead of five hours, it will take seven hours to get to Sydney.'

A little later, the pilot told the passengers that a second engine had failed. 'But we still have two engines left. We’re still fine, except now it will take ten hours to get to Sydney.'

Somewhat later, the pilot again came on the intercom and announced that a third engine had died. 'But never fear, because this plane can fly on a single engine. Of course, it will now take 18 hours to get to Sydney.'

At this point, one statistician turned to another and said, 'Gee, I hope we don't lose that last engine, or we'll be up here forever!'
Technology 7.1  The Quadratic Formula and the Calculator
You can be more efficient and accurate in the use of your calculator.

Example:  Solve:  $5x^2 - 2x - 4 = 0$

1 Write the quadratic  $5x^2 - 2x - 4 = 0$
2 Write the general quadratic  $ax^2 + bx + c = 0$
3 Decide values of $a$, $b$, $c$  $a=5 \quad b=-2 \quad c=-4$
4 Write the quadratic formula  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
5 Substitute for $a$, $b$, and $c$  
   
   
   $x = \frac{-2 \pm \sqrt{(-2)^2 - 4 \times 5 \times -4}}{2 \times 5}$

6 Calculate

   
   $x = 1.12 \quad \text{or} \quad x = -0.72$

To calculate this in one go.

\[ \sqrt{(-2)^2 - 4 \times 5 \times -4} \]

you need an extra set of brackets.

\[ \sqrt{(-2)^2 - 4 \times 5 \times -4} \]

to give the answer 9.17

Technology 7.2  Quadratics and the Graphics Calculator
Use a graphics calculator to check your answers to Exercises 7.4, 7.5, 7.6, and 7.7.

Example:  Graph:  $y = x^2 + x - 6$  and solve:  $x^2 + x - 6 = 0$

Press  \( Y = \)  and enter the function eg.,  $x^2 + x - 6$

Press  \( \text{Graph} \)  to see a graph of the function.

To solve:  $x^2 + x - 6 = 0$  is to find the x-intercepts.

a) Use \( \text{CALC} \)  to find the intercepts (some calculators use zero and value).

b) Use \( \text{TRACE} \)  and move the cursor to the x-intercepts.

c) Use \( \text{TABLE} \)  to find the x-intercept (where $y = 0$).

Can you get the answers:  $x = -3 \quad \text{and} \quad x = 2$
## Exercise 7.13

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>$3(x - 4) = 15$</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>$-2(x - 3) = 8$</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>$7(2x - 1) = 14$</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>$2(x - 3) + 3x = 11$</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>$5(x - 3) - 2x = 12$</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>$6x - 8 = 2x + 4$</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>$2x + 2 = 16 - 5x$</td>
<td></td>
</tr>
</tbody>
</table>

11 The sum of the interior angles of a polygon is given by the formula: $S = 90(2n - 4)$, where $n$ is the number of sides on the polygon. How many sides in a polygon with an interior angle sum of $540^\circ$?

12 The perimeter of a block of land is 110 m. The length is 21 m longer than the breadth. Find the length and the breadth.

<p>| | | |</p>
<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>13</td>
<td>$x^2 + 4x + 4 = 0$</td>
<td>16</td>
</tr>
<tr>
<td>14</td>
<td>$x^2 + 10x + 25 = 0$</td>
<td>17</td>
</tr>
<tr>
<td>15</td>
<td>$x^2 - 2x + 1 = 0$</td>
<td>18</td>
</tr>
<tr>
<td>19</td>
<td>$x^2 + 5x - 6 = 0$</td>
<td>20</td>
</tr>
<tr>
<td>21</td>
<td>$x^2 - 5x - 6 = 0$</td>
<td>22</td>
</tr>
<tr>
<td>23</td>
<td>$x^2 - 10x + 9 = 0$</td>
<td>24</td>
</tr>
</tbody>
</table>

25 $x^2 + 5x + 2 = 0$

### Given the quadratic: $ax^2 + bx + c = 0$

### The solution is:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

26 $3x^2 + 7x - 3 = 0$

27 $x^2 - 11x + 3 = 0$

28 $-4x^2 - 3x + 1 = 0$

29 A stone is thrown vertically with a velocity of 40 m/s. The motion of the stone is described by the relationship: $5t^2 - 40t + h = 0$, where $h$ is the height in metres and $t$ is the time in seconds. How long will it take the stone to reach a height of 10 m?

30 Calculate $x$ in the following figure:
Exercise 7.14

1. \(2(x - 3) = 10\)
2. \(-5(x - 4) = 10\)
3. \(4(3x - 1) = 20\)
4. \(3(x - 1) + 2x = 12\)
5. \(7(x - 2) - 3x = 4\)
6. \(7x - 7 = 2x + 3\)
7. \(2x + 2 = 13 - 2x\)

Check: \((\frac{10 + 3}{5}) = \frac{13}{5} = 2\)

8. \(\frac{x}{5} - 2 = -5\)
9. \(\frac{3x - 2}{2} = 5\)
10. \(\frac{2x - x}{3} = 5\)

11. The volume, \(V\), of a square based prism is given by the formula: \(V = w^2h\), where \(w\) is the width of the base and \(h\) is the height of the prism. If the width of the prism is 6 cm and the volume is 162 cm³, what is the height of the prism?

12. An isosceles triangle has two angles each twice the size of the third angle. What is the size of each angle?

\[x^2 - 8x + 16 = 0\]
\[(x - 4)^2 = 0\] \{perfect square\}
\[x - 4 = 0\] \{square root both sides\}
\[x = 4\] \{inverse of \(-4\)\}

Check: \((4)^2 - 8\times4 + 16 = 0\)

\[x^2 - 3x - 10 = 0\]
\[x^2 - 5x + 2x - 10 = 0\] \{grouping pairs\}
\[x(x - 5) + 2(x - 5) = 0\] \{factorising\}
\[(x - 5)(x + 2) = 0\] \{factorising\}

Either \(x - 5 = 0\) or \(x + 2 = 0\)
\[x = 5\] or \(x = -2\)

Check: \((5)^2 - 4\times5 - 10 = 0\)
Check: \((-2)^2 - 4\times(-2) - 10 = 0\)

13. \(x^2 + 6x + 9 = 0\)
14. \(x^2 + 8x + 16 = 0\)
15. \(x^2 - 4x + 4 = 0\)
16. \(x^2 - 10x + 25 = 0\)
17. \(x^2 + 4x + 3 = 0\)
18. \(x^2 + 6x + 8 = 0\)
19. \(x^2 + x - 6 = 0\)
20. \(x^2 - 3x - 10 = 0\)
21. \(x^2 - 5x - 6 = 0\)
22. \(x^2 - x - 12 = 0\)
23. \(x^2 - 9x + 8 = 0\)
24. \(x^2 - 13x + 12 = 0\)

25. \(x^2 + 3x + 1 = 0\)
26. \(3x^2 + 5x - 2 = 0\)
27. \(x^2 - 7x + 5 = 0\)
28. \(-2x^2 - x + 1 = 0\)

Given the quadratic: \(ax^2 + bx + c = 0\)
The solution is:
\[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]

29. A stone is thrown vertically with a velocity of 50 m/s. The motion of the stone is described by the relationship: \(5t^2 - 50t + h = 0\), where \(h\) is the height in metres and \(t\) is the time in seconds. How long will it take the stone to reach a height of 40 m?

30. Calculate \(x\) in the following figure:

\[\text{Area} = 4\]
\[\frac{x + 1}{x} = 1\]