

Trigonometry 1

12

Measurement and Geometry → Pythagoras and Trigonometry

- ★ Use similarity to investigate the constancy of the sine, cosine and tangent ratios for a given angle in right-angled triangles.
- ★ Develop an understanding of the relationship between the corresponding sides of similar right-angled triangles.
- ★ Apply trigonometry to solve right-angled triangle problems.
- ★ Understand the terms 'adjacent' and 'opposite' sides in a right-angled triangle.

And with a torch it even works at night.



A TASK

A sundial measures the time using the position of the sun. Use the Internet to find a sundial design (there are hundreds of different designs).

- Make your sundial.
- Test your sundial.
- Demonstrate your sundial to your class.

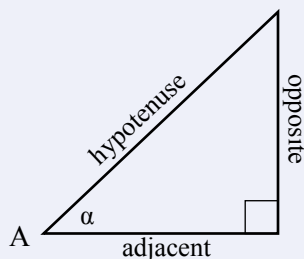
A LITTLE BIT OF HISTORY

The Babylonians, around 3 000 years ago, measured angles in degrees, minutes, and seconds.

Around 2 000 years ago, Indian astronomers developed trigonometry based on a sine function. The Indian sine function was the length of the opposite side for a given hypotenuse.

Muslim scientists had tables for sine and tangent that were extremely accurate (1 part in 700 million).

When calculus was invented, around 300 years ago, trigonometric functions became much more important in many more pure and applied mathematical applications.



$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$

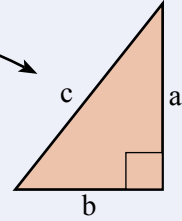
Pythagoras' Theorem

In any right-angled triangle:

The square on the hypotenuse is equal to the sum of the squares on the other two sides.

$$c^2 = a^2 + b^2$$

The hypotenuse is the longest side.
It is opposite the right-angle (90°).



Exercise 12.1

Find the length of the hypotenuse in each of the following:

First add a, b, c

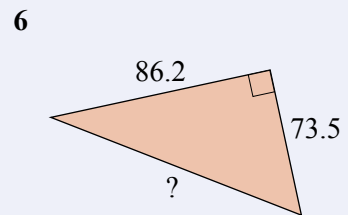
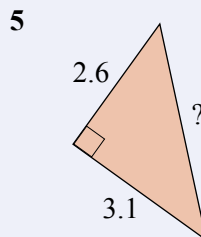
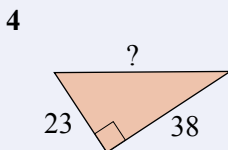
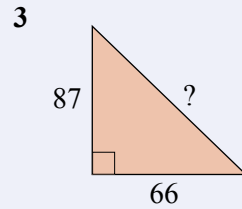
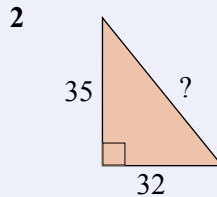
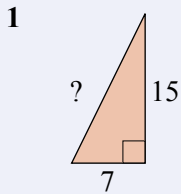
$$c^2 = a^2 + b^2$$

$$c^2 = 53^2 + 47^2$$

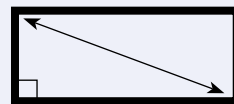
$$c^2 = 5018$$

$$c = \sqrt{5018}$$

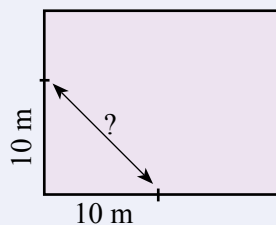
$$c = \underline{70.84}$$



- 7 A 3m wide by 1.4 m rectangular gate needs a diagonal brace to keep it rigid.
What should be the length of the diagonal?



- 8 A builder checks the right-angle of a slab corner by making marks 10 m out from each corner.
How far apart should the marks be?

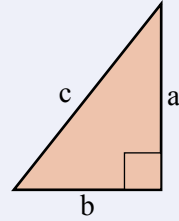


Pythagoras' Theorem

In any right-angled triangle:

The square on the hypotenuse is equal to the sum of the squares on the other two sides.

$$a^2 + b^2 = c^2$$



Exercise 12.2

Find the length of the unknown in each of the following:

First add a, b, c

$$a^2 + b^2 = c^2$$

$$a^2 + 5.1^2 = 6.4^2$$

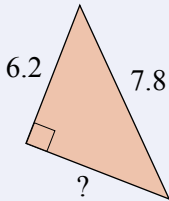
$$a^2 = 6.4^2 - 5.1^2$$

$$a^2 = 14.95$$

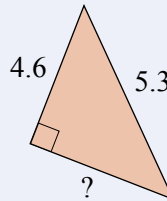
$$a = \sqrt{14.95}$$

$$a = \underline{3.87}$$

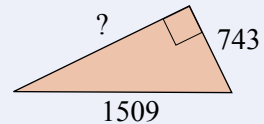
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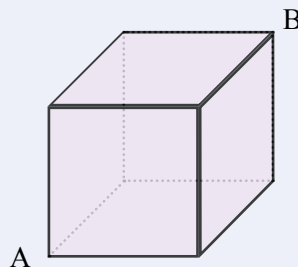
Rounding to two decimal places, first look at the third decimal place:

56.231694	27.01769	1.07276	4.79634216
↑	↑	↑	↑
less than 5 thus 56.23	5 or more thus 27.02	less than 5 thus 1.07	5 or more thus 4.80

- 4 A 45 m tower is supported by guy wires. The guy wires are attached to the top of the tower and anchored to the ground out from the tower. If the guy wires are 150 m long, how far out from the tower are they anchored?

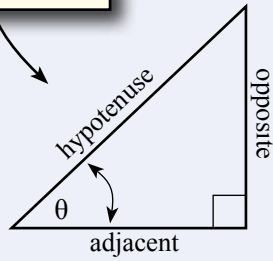


- 5 What is the distance from A to B on the 10 cm cube?



Naming Sides

The **hypotenuse** is the longest side. It is opposite the right-angle.



Opposite is 'opposite' the angle.

Adjacent means 'next to' the angle.

Small letters from the Greek alphabet are sometimes used to represent angles.

Alpha	α
Beta	β
Theta	θ
Phi	ϕ
Lamda	λ

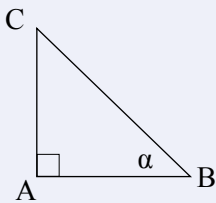
Exercise 12.3

For each of the following triangles, name

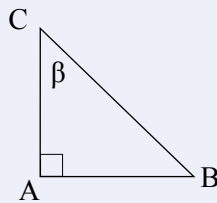
- the hypotenuse.
- the side adjacent to the angle.
- the side opposite the angle.

<p>AC is the hypotenuse. BC is adjacent. AB is opposite.</p>	<p>AB is the hypotenuse. AC is adjacent. BC is opposite.</p>
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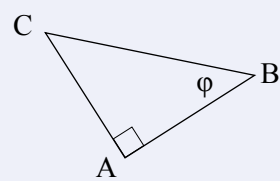
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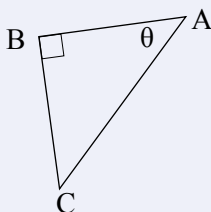
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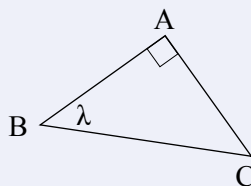
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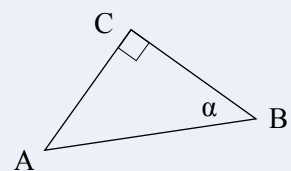
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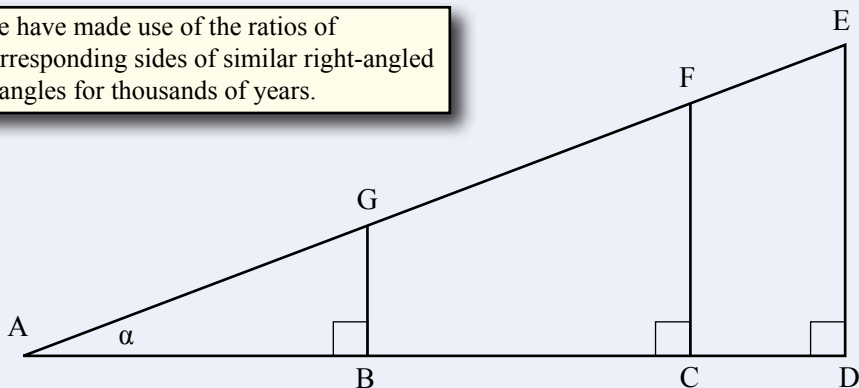


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Trigonometry

We have made use of the ratios of corresponding sides of similar right-angled triangles for thousands of years.



You will need a ruler to measure each line.

Exercise 12.4

1 Copy and complete the following table:

Triangle	Opposite	Adjacent	$\frac{\text{opposite}}{\text{adjacent}}$
$\triangle ABG$	1.7 cm	4.6 cm	$1.7 \div 4.6 = 0.37$
$\triangle ACF$			
$\triangle ADE$			

$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$

$$\alpha = 20^\circ$$

Use your calculator:

$$\boxed{\tan} \boxed{20} \boxed{=}$$

2 Complete the following table:

Triangle	Opposite	Hypotenuse	$\frac{\text{opposite}}{\text{hypotenuse}}$
$\triangle ABG$	1.7 cm	4.9 cm	$1.7 \div 4.9 = 0.35$
$\triangle ACF$			
$\triangle ADE$			

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\alpha = 20^\circ$$

Use your calculator:

$$\boxed{\sin} \boxed{20} \boxed{=}$$

3 Complete the following table:

Triangle	Adjacent	Hypotenuse	$\frac{\text{adjacent}}{\text{hypotenuse}}$
$\triangle ABG$	4.6 cm	4.9 cm	$4.6 \div 4.9 = 0.94$
$\triangle ACF$			
$\triangle ADE$			

$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\alpha = 20^\circ$$

Use your calculator:

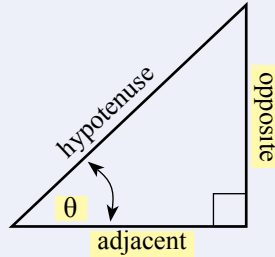
$$\boxed{\cos} \boxed{20} \boxed{=}$$

The Tan Ratio

Trigonometry was developed thousands of years ago to solve the many problems in surveying, engineering, architecture, astronomy, etc, etc, etc.

Trigonometry n. branch of mathematics dealing with the relationships between angles and sides of triangles.

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$



Exercise 12.5

Find $\tan \alpha$ and the size of the angle α .

3 is opposite α



4 is adjacent to α

$$\tan \alpha = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan \alpha = \frac{3}{4} = 0.75$$

$$\alpha = \tan^{-1} 0.75$$

$$\alpha = 36.9^\circ$$

Use your calculator:

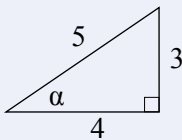
2ndF tan⁻¹ 0.75 =

\tan^{-1} means 'an angle whose tan is'.

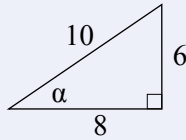
Thus $\tan^{-1} 0.75$ means 'an angle whose tan is 0.75' (which is 36.9°).

Make sure your calculator is on **degrees**.

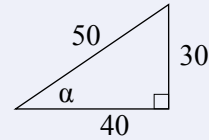
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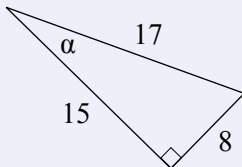
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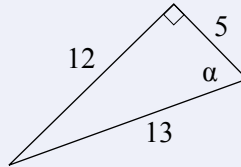
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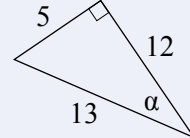
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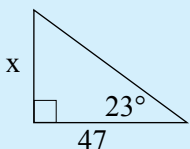
The Tan Ratio

Trigonometry can be used to find a side after knowing a side and an angle in a right-angled triangle.

Trigonometry is used millions and millions of times every day.

Exercise 12.6

Find x in each of the following right-angled triangles:



$$\tan \alpha = \frac{\text{opposite}}{\text{adjacent}}$$

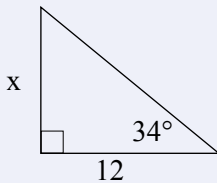
$$\tan 23 = \frac{x}{47}$$

$$\tan 23 \times 47 = x \quad \{\text{inverse of } \div \text{ is } \times\}$$

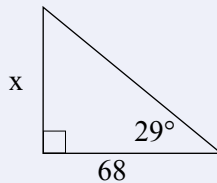
$$\underline{19.95 = x}$$

Make sure your calculator is in degrees (deg).

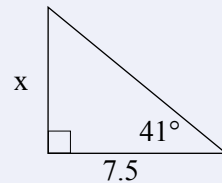
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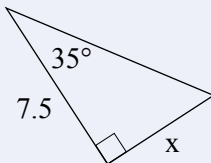
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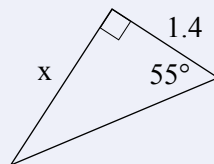
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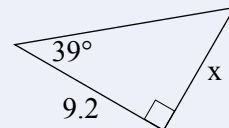
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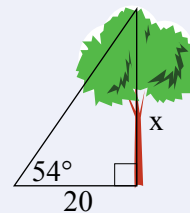


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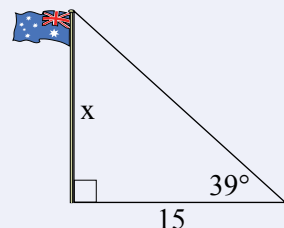
7

20 m out from the base of a tree, a clinometer measures the angle of elevation to the top of the tree as 54° . Find the height of the tree.



8

15 m out from the base of a flagpole, a clinometer measures the angle of elevation to the top of the flagpole as 39° . Find the height of the flagpole.



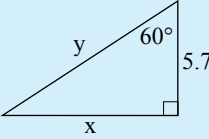
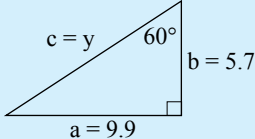
The Tan Ratio

Trigonometry can be used to find a side after knowing a side and an angle in a right-angled triangle.

Pythagoras' Theorem can be used to find the third side after knowing two sides in a right-angled triangle.

Exercise 12.7

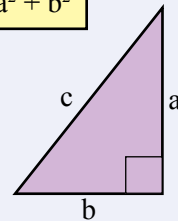
Find the unknown sides:

 <p style="text-align: center;">$\tan \alpha = \frac{\text{opposite}}{\text{adjacent}}$</p> <p style="text-align: center;">$\tan 60 = \frac{x}{5.7}$</p> <p style="text-align: center;">$\tan 60 \times 5.7 = x$</p> <p style="text-align: center;"><u>$9.87 = x$</u></p>	 <p style="text-align: center;">$c^2 = a^2 + b^2$</p> <p style="text-align: center;">$y^2 = 9.9^2 + 5.7^2$</p> <p style="text-align: center;">$y^2 = 130.5$</p> <p style="text-align: center;">$y = \sqrt{130.5}$</p> <p style="text-align: center;"><u>$y = 11.40$</u></p>
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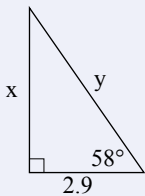
In any right-angled triangle:

The square on the hypotenuse is equal to the sum of the squares on the other two sides.

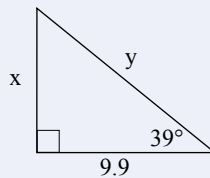
$$c^2 = a^2 + b^2$$



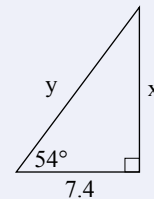
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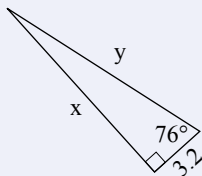
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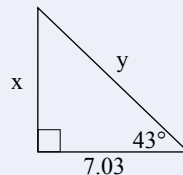
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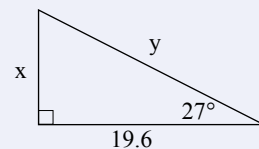
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5



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- 7 A student with a clinometer, is lying on the ground 4.6 m out from the base of a flagpole. If the clinometer reads 45° , what is the height of the flagpole?
- 8 6.2 m out from the base of a tree, a clinometer measures the angle of elevation to the top of the tree as 34° . Find the height of the tree.
- 9 4.3 m out from the base of a building, a clinometer measures the angle of elevation to the top of the building as 45° . Find the height of the building.
- 10 The angle of elevation of the top of a tower from a point 37 m out from the base of the tower is 53° . Find the height of the tower correct to one decimal place.

The Tan Ratio

Trigonometry can be used to find an **angle** after knowing two sides in a right-angled triangle.

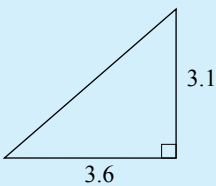
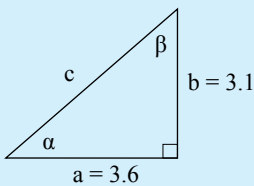
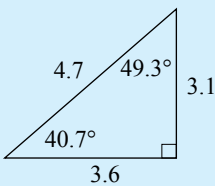
The angles in a triangle sum to 180°

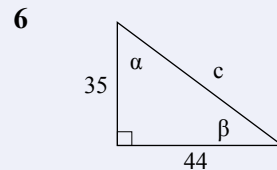
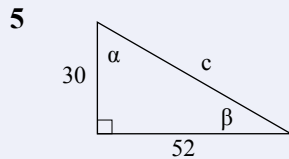
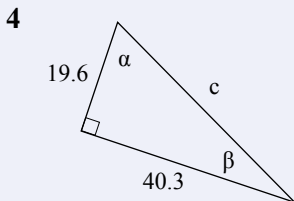
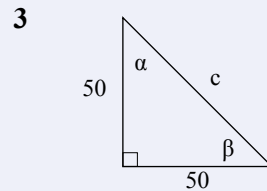
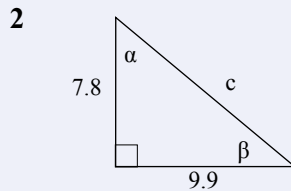
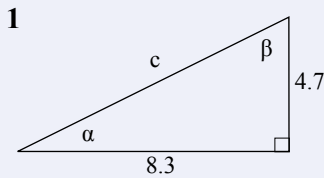
Pythagoras' Theorem can be used to find the third side after knowing two sides in a right-angled triangle.

Solve means 'find **all** unknowns'.
A triangle has **3 sides**
and **3 angles**.

Exercise 12.8

Solve the following triangles:

		
$\tan \alpha = \frac{\text{opposite}}{\text{adjacent}}$ $\tan \alpha = \frac{3.1}{3.6}$ $\alpha = \tan^{-1}(3.1 \div 3.6)$ $\alpha = 40.7^\circ$	$c^2 = a^2 + b^2$ $c^2 = 3.6^2 + 3.1^2$ $c^2 = 22.57$ $c = \sqrt{22.57}$ $c = 4.75$	$\text{sum of angles} = 180^\circ$ $\alpha + \beta + 90 = 180$ $40.7 + \beta + 90 = 180$ $\beta = 180 - 40.7 - 90$ $\beta = 49.3^\circ$



$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

The **tangent ratio** is one of several ratios involving the relationships between the sides and angles of triangles. Sin and cos are in Chapter 17.

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$$

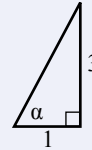
$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$$

Mental Computation

Mental computation gives you practice in thinking.

Exercise 12.9

- 1 Spell Trigonometry.
- 2 What is the tan ratio?
- 3 In the triangle, what is $\tan\alpha$?
- 4 If one angle in a right-angled triangle is 30° , what is the third angle?
- 5 Two sides in a right-angled triangle are 1 and 3. Hypotenuse?
- 6 Write in scientific notation: 54 000
- 7 Write in scientific notation: 0.003 2
- 8 $10^6 \div 10^4$
- 9 What is the average of 2, 2, 3, 4, 5?
- 10 16×25



$$c^2 = 1^2 + 3^2$$

$$c^2 = 1 + 9$$

$$c = \sqrt{10}$$

If you can dream it, you can do it - Walt Disney

$$16 \times 25$$

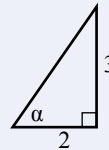
$$= 4 \times 4 \times 25$$

$$= 4 \times 100$$

$$= 400$$

Exercise 12.10

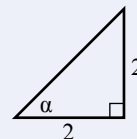
- 1 Spell Tangent.
- 2 What is the tan ratio?
- 3 In the triangle, what is $\tan\alpha$?
- 4 If one angle in a right-angled triangle is 60° , what is the third angle?
- 5 Two sides in a right-angled triangle are 2 and 3. Hypotenuse?
- 6 Write in scientific notation: 170 000
- 7 Write in scientific notation: 0.000 14
- 8 $10^6 \div 10^3$
- 9 What is the average of 1, 2, 3, 4, 5?
- 10 20×25



All of us could take a lesson from the weather. It pays no attention to criticism..

Exercise 12.11

- 1 Spell Pythagoras.
- 2 What is the tan ratio?
- 3 In the triangle, what is $\tan\alpha$?
- 4 If one angle in a right-angled triangle is 40° , what is the third angle?
- 5 Two sides in a right-angled triangle are 2 and 2. Hypotenuse?
- 6 Write in scientific notation: 3 000 000
- 7 Write in scientific notation: 0.000 000 9
- 8 $10^9 \div 10^6$
- 9 What is the average of 2, 3, 3, 4?
- 10 24×25



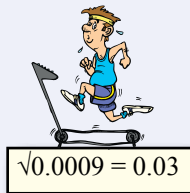
Conveyancers compile the documentation needed for the sale and purchase of real estate.

- Relevant school subjects are English and Mathematics.
- Courses usually involve a diploma or business degree.

Competition Questions

Exercise 12.12

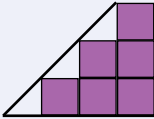
- 1 What is the square root of 400?
- 2 What is the square root of 4?
- 3 What is the square root of 0.04?
- 4 What is the square root of 0.0004?
- 5 What is the gradient of the ramp, the thick line, in each of the following?



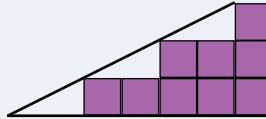
Build maths muscle and prepare for mathematics competitions at the same time.

gradient=slope=tangent ratio.
Assume each block is square.

a)



b)



c)



- 6 One angle in a right-angled triangle is 37° , what is the size of the other two angles?
- 7 Two sides of a triangle are 6 cm and 3 cm. Can the third be 10 cm?
- 8 Two sides of a triangle are 6 cm and 3 cm. Can the third be 2 cm?

A right-angled isosceles triangle has an area of 18. What is the length of the hypotenuse?

$$\text{area} = \frac{\text{base} \times \text{height}}{2}$$

$$18 = \frac{x^2}{2}$$

$$36 = x^2$$

$$6 = x$$

$$c^2 = a^2 + b^2$$

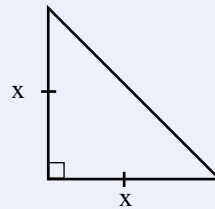
$$c^2 = 6^2 + 6^2$$

$$c^2 = 72$$

$$c = \sqrt{72}$$

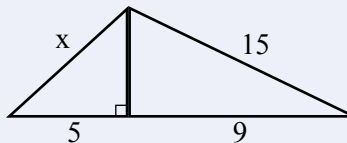
$$c = \sqrt{(36 \times 2)}$$

$$c = \underline{6\sqrt{2}}$$



An isosceles triangle has two equal sides.

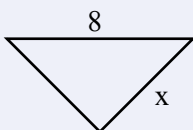
- 9 A right-angled isosceles triangle has an area of 50. What is the length of the hypotenuse?
- 10 A right-angled isosceles triangle has an area of 32. What is the length of the hypotenuse?
- 11 Find the value of x in the following diagram:



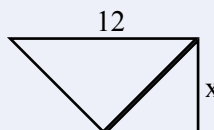
An isosceles triangle has two equal angles of 45° opposite the equal sides.

- 12 All angles in the following diagrams are either 45° or 90° . Find x .

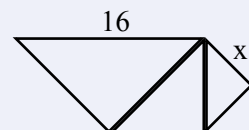
a)



b)



c)



A Couple of Puzzles

Exercise 12.13

1 Complete the following:

$$1 + 3 + 5 =$$

$$1 + 3 + 5 + 7 =$$

$$1 + 3 + 5 + 7 + 9 =$$

$$1 + 3 + 5 + 7 + 9 + 11 =$$

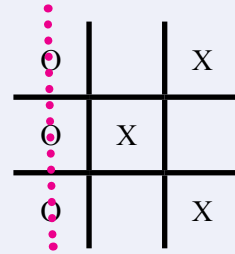
$$1 + 3 + 5 + 7 + 9 + 11 + 13 =$$

$$1 + 3 + 5 + 7 + \dots + 97 + 99 =$$

A Game

Diox is a two player game based on naughts and crosses.

The winner is the first person to have three Os or three Xs in a row, column, or diagonal as in the original game of naughts and crosses.



Players take turns throwing a die.

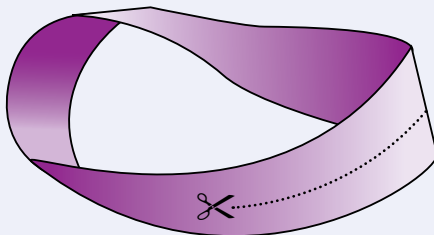
- an even number means the player must place an X
- an odd number means the player must place an O

A Sweet Trick

The Mobius strip

- 1 Obtain a long strip of paper that is about 5 cm wide.
- 2 Make a loop with a half twist and tape the two ends together.
- 3 Ask your audience what they would expect if you cut the strip of paper in half along the middle of the strip of paper.
- 4 Cut along the middle of the loop and produce a larger loop and not two loops as would be expected.

- What happens if you cut along the middle of the larger loop again?
- What happens if you cut a loop with a full twist?



Why did the chicken cross the Mobius strip?
To get to the same side.

The Mobius strip is the source of a number of puzzles based on the half twist making the inside surface and the outside surface the same.



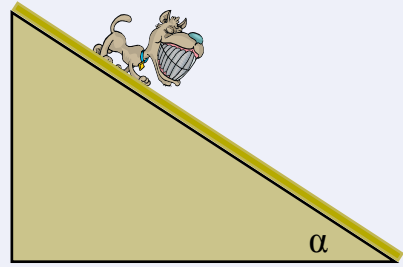
Try it by drawing a line along the outside that is also the inside.

Investigations

Investigation 12.1 Slope

The tan ratio is used to measure slope or gradient.

- 1 What is the angle at which an object begins to slip down the slope (Use the tan ratio to calculate the angle)?
- 2 Compare this angle with other objects.
- 3 Why the difference?



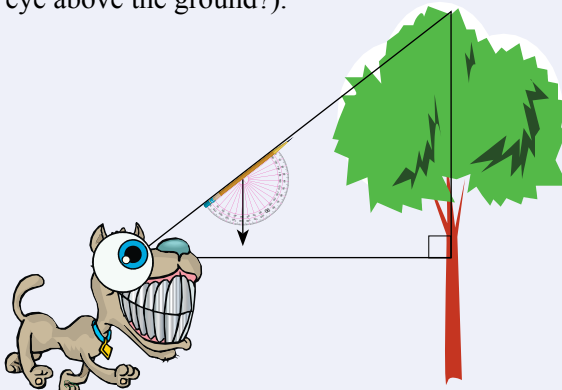
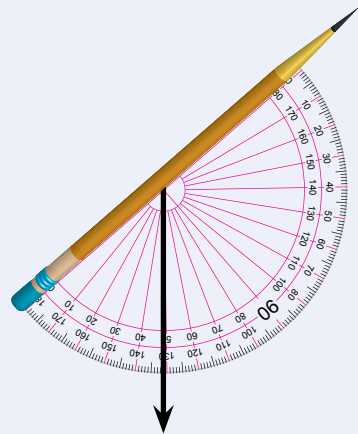
Investigation 12.2 Natural Slope?

Investigate

The angle of natural slopes.

Investigation 12.3 Find Heights

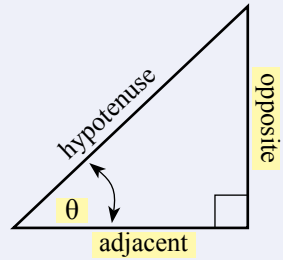
- 1 Make a clinometer using a straw, a protractor, a small weight on the end of a string, and sticky tape.
- 2 Measure a distance out from the base of a tree or flagpole.
- 3 Aim the clinometer at the top of the tree and measure the angle of elevation.
- 4 Use the tan ratio to calculate the height of the tree (What about the height of your eye above the ground?).



Technology

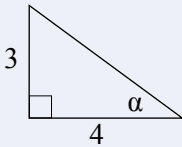
Use a spreadsheet to solve the previous exercises.

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$



Technology 12.1 The Tan Ratio and the Spreadsheet

a) Given the opposite and adjacent, find the angle.

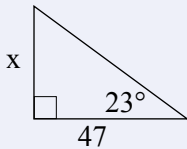


	a	b	c	d
1	Opposite	Adjacent	Tan α	α
2	3	4	0.75	36.87

Enter the formula:
=atan(c2)*180/pi()

The *180/pi() is needed to convert radians to degrees.

b) Given the angle and adjacent, find the opposite.



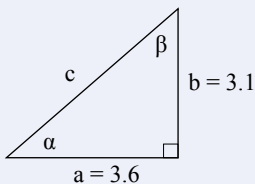
	a	b	c
1	Angle	Adjacent	Opposite
2	23	47	19.95

Enter the formula:
=tan(a2*pi()/180)*b2

The *pi()/180 is needed to convert degrees to radians.

a) Given the opposite and adjacent, solve the triangle.

	a	b	c	d	e
1	Opposite	Adjacent	α	β	Hypotenuse
2	3.1	3.6	40.73	49.27	4.75



Enter the formula:
=atan(a2/b2)*180/pi()

The *180/pi() is needed to convert radians to degrees.

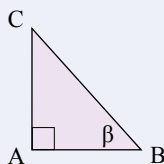
Enter the formula:
=sqrt(a2*a2+b2*b2)

Enter the formula:
=180-90-c2

Chapter Review 1

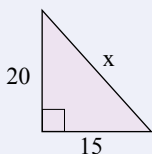
Exercise 12.14

- 1 For the adjacent triangle, name:
- the hypotenuse.
 - the side adjacent to the angle.
 - the side opposite the angle.

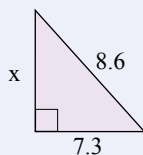


- 2 Use Pythagoras' Theorem to find the unknown:

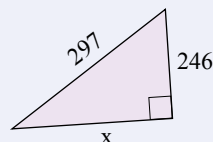
a)



b)

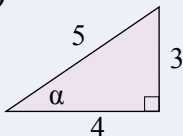


c)

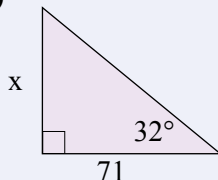


- 3 Find the unknown in each of the following triangles:

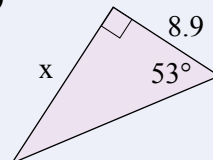
a)



b)

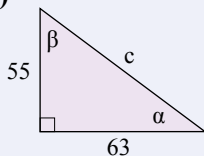


c)

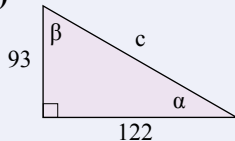


- 4 Solve the following triangles:

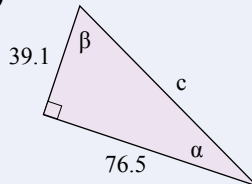
a)



b)

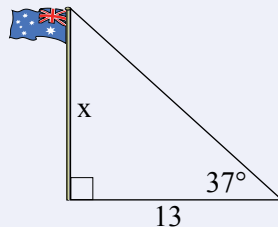


c)

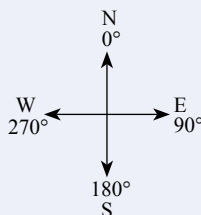


Solve means 'find all unknowns'!

- 5 13 m out from the base of a flagpole, a clinometer measures the angle of elevation to the top of the flagpole as 37° . Find the height of the flagpole.



- 6 A ship sails due north for 15 km, then on a bearing of 160° until the ship is due east of its starting point. How far is the ship from its starting point?

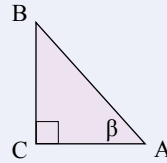


Chapter Review 2

Exercise 12.15

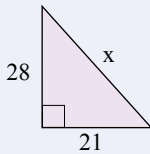
1 For the adjacent triangle, name:

- a) the hypotenuse.
- b) the side adjacent to the angle.
- c) the side opposite the angle.

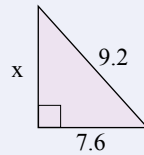


2 Use Pythagoras' Theorem to find the unknown:

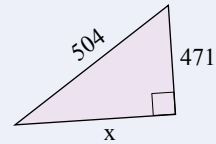
a)



b)

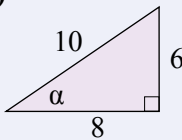


c)

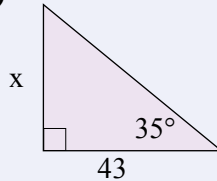


3 Find the unknown in each of the following triangles:

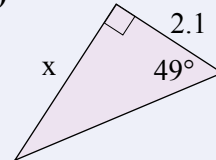
a)



b)

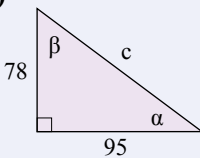


c)

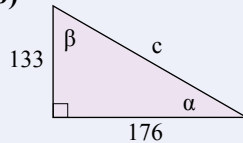


4 Solve the following triangles:

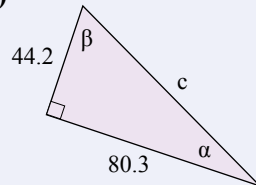
a)



b)

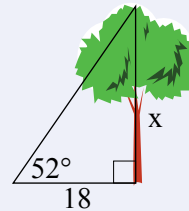


c)



Solve means 'find all unknowns'!

5 18 m out from the base of a tree, a clinometer measures the angle of elevation to the top of the tree as 52° . Find the height of the tree.



6 A ship sails due East for 60 km, then on a bearing of 225° until the ship is due south of its starting point. How far is the ship from its starting point?

